Time: 3 hrs **Marks: 80**

Note :

1) **Q. No. 01 is compulsory.**
2) **Solve any three from Q. No. 02 to 06.**
3) **Numbers to the right indicate full marks.**
4) **Use of statistical tables is allowed.**

Q. 1. Solve.

a) If $A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$ find the sum and product of Eigen values A. 5

b) Integrate the function $f(z) = z^2$ from A(0, 0) to B(1, 1) along straight line AB. 5

c) Find the Z-Transform of $(k) = a^k$, k < 0. 5

d) A transmission channel has a per-digit error probability p = 0.01. Calculate the probability of more than 1 error in 10 received digits using Poisson distribution. 5

Q. 2.

a) Find the Eigenvalues and Eigenvectors of the matrix $A = \begin{bmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix}$. 6

b) Find the Z-Transform of Cos $\left(\frac{k\pi}{4} + \alpha\right)$ $k \geq 0$. 6

c) Use the dual simplex method to solve the LPP
Min.. $Z = 2X_1 + 2X_2 + 4X_3$
$2X_1 + 3X_2 + 5X_3 \geq 2$, $3X_1 + X_2 + 7X_3 \leq 3$, $X_1 + 4X_2 + 6X_3 \leq 5$ $X_1, X_2, X_3 \geq 0$ 8

Q. 3.

a) Evaluate $\int_C \frac{z^2}{(z-1)(z-2)} dz$ Where C is a circle $|z-1|=1$. 6

b) Verify Caley-Hamilton theorem and hence find A^{-1} and A^4 where $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 4 \\ 3 & 1 & -1 \end{bmatrix}$. 6

c) Solve the LPP by Big -M method
Maximize $Z = 3X_1 - 2X_2$
subject to $2X_1 + X_2 \leq 2$, $X_1 + 3 \geq 3$, $X_1, X_2, \geq 0$. 8

Q. 4.

a) Find inverse Z transform of $F(z) = \frac{1}{(z-1)(z-3)}$ for i) $|z| < 1$, ii) $1 < |z| < 3$. 6

b) The following data represent the marks obtained by 12 students in two tests, one held before the coaching and the other after the coaching.

Test I : 55, 60, 65, 75, 49, 25, 18, 30, 35, 51, 61, 72. 6

Test II : 63, 70, 70, 81, 54, 29, 21, 38, 32, 50, 70, 80.

Do the data indicate that the coaching was effective in improving the performance of the students?

c) Find all possible Laurent's series expansions of the function $f(z) = \frac{1}{(z-1)(z+2)}$ about z = 0 indicating the region of convergence in each case. 8

Q. 5.

a) Determine all basic solutions to the following problem

Max. $Z = x_1 - 2x_2 + 4x_3$ 6

$x_1 + 2x_2 + 3x_3 = 7$, $3x_1 + 4x_2 + 6x_3 = 15$, $x_1, x_2, x_3 \geq 0$.

b) Using Normal distribution, find the probability of getting 55 heads in the toss of 100 fair coins. 6

c) Solve the NLPP

Optimize $Z = 10x_1 + 8x_2 + 6x_3 + 2{x_1}^2 + {x_2}^2 + 3{x_3}^2 - 100$

Subject to $x_1 + x_2 + x_3 = 20$, $x_1, x_2, x_3 \geq 0$. 8

Q. 6.

a) Show that the given matrix is diagonalizable and hence find diagonal form and transforming matrix where $A = \begin{bmatrix} 4 & 6 & 6 \\ 1 & 3 & 2 \\ -1 & -5 & -2 \end{bmatrix}$. 6

b) Of the 64 off springs of a certain cross between guinea pigs 34 were red, 10 were black and 20 were white. According to the generic model these numbers should be in the ratio 9 : 3 : 4. Use 2- test to check whether the data are consistent with the model. 6

c) Max. $Z = 4x_1 + 6x_2 - {x_1}^2 - {x_2}^2 - {x_3}^2$, Subject to $x_1 + x_2 \leq 2$ and $2x_1 + 3x_2 \leq 12$, $x_1, x_2 \geq 0$ by K-T condition. 8

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